existence indicates that the air which inflates the bubble is processed by the weak shock and does not come from the adjacent subsonic region. Besides, the pressure measurements reported in this section reveal the supersonic character of this jet or shear layer.

After the optical detection of the jet, we performed sublimation tests under flow conditions similar to those of Fig. 2. It was found that a dark ring was formed at the white surface of the cylinder, exactly where the annular supersonic jet impinges.

The dynamic characteristics of the pulsating flow were also detected by the pressure measurements. The measured amplitude of the pressure fluctuation at the face of the afterbody at $r/R=0.25$ vs the spike length is shown in Fig. 4. The flow conditions were $M_\infty=6$ and $p_t=20$ bar. We observe that the amplitude of the fluctuation is independent of spike length for the pulsation mode and equal to $\Delta p=1$ bar. If we normalize this value by the pressure p_3 behind the strong normal shock AB, we find that $\Delta p/p_3=2.0$. This simply shows that the fluctuation of the pressure in the separation region is greater than the pressure behind the strong shock and, consequently, that the air which fills the dead air region could not have come from the adjacent subsonic region but actually comes through the conical shock wave and is compressed at the impingement region.

IV. Indirect Evidence

According to the mechanism of the pulsation described in Sec. II, the rapid inflation of the separation bubble during the impulsive start of the flow co-occurs with the movement of the shock intersection away from the body. It is obvious that if in this phase the foreshock covers the afterbody before a sufficient quantity of high-pressure air is trapped in the bubble, then the explosive expansion of the bubble will not occur and consequently the pulsation mode will not be established.

In order to prove this hypothesis and thus provide indirect evidence for the validity of the proposed mechanism, we have studied experimentally the parameters which affect the development of the initial phase of the flow, i.e., the volume of the axisymmetric cavity and the inviscid position of the shock intersection.

Effect of the Volume of the Axisymmetric Cavity

If our hypothesis is valid, by increasing the volume of the axisymmetric cavity a limit must be reached above which the pulsation mode will not appear. One way of increasing the volume is to increase the length of the forebody. The effect of this parameter is shown in Fig. 4 for the case of the spiked cylinder. It is observed that indeed there exists a limit $(1/d>1.4)$ above which the pulsation turns into oscillation.

As additional proof of the effect of the volume we mention a special test performed by Loll[9] at VKI. Varying the dead-air region volume of a spiked flat cylinder by connecting it through a concentric hole with the inner part of the (hollow) cylinder, Loll found that for sufficient cylinder volume the flow became steady instead of pulsating.

Also a review of the experimental studies shows that if a concave body is in sufficient incidence, the pulsation mode does not appear. This happens because the high-pressure air escapes leeward.

Effect of the Position of the Shock Intersection

According to our hypothesis the nearer the shock intersection lies to the surface and to the shoulder of the afterbody, the faster it will be pushed radially outward and the smaller the quantity of the trapped air will be, providing fewer possibilities for the occurrence of pulsation. The effect of this parameter was studied by using flat-ended cylinders equipped with spikes of constant diameter and conical tips of variable angle. Testing step by step we measured the spike length for which the pulsation turns into oscillation. The results are plotted in Fig. 5 and confirm the hypothesis of the importance of the initial position of the shock intersection in determining the type of the flow.

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Asymptotic Properties of the Zarf

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THE Orr-Sommerfeld equation for a plane parallel flow

$$(u(z),0,0) \tag{1}$$

is

$$\psi'''' - 2(\alpha^2+\beta^2)\psi'' + (\alpha^2+\beta^2)^2\psi = iR\{(\alpha u-\omega)[\psi''-(\alpha^2+\beta^2)\psi]-\alpha u''\psi\} \tag{2}$$

together with the boundary conditions $\psi(0)=\psi'(0)=0$ and $\psi\to\infty$ as $z\to\infty$ is obtained by assuming that small disturbances to the basic flow can exist in the form

$$\exp[i(\alpha x+\beta y-\omega t)]\psi(z) \tag{3}$$

where R is a suitably defined local Reynolds number. This

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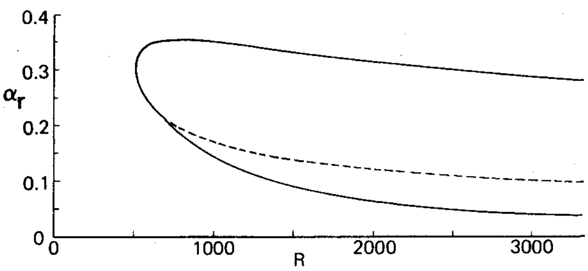

Fig. 1a Projection of the zarf on α_r, R plane for Blasius flow (dashed line is two-dimensional neutral curve).

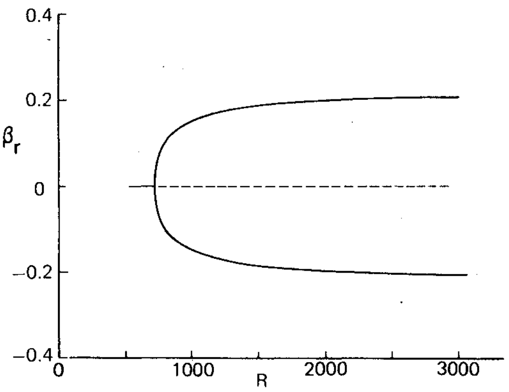

Fig. 1b Projection of the zarf on β_r, R plane for Blasius flow.

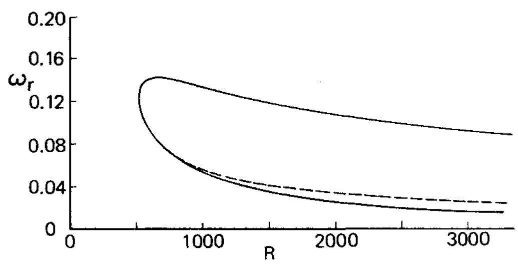

Fig. 1c Projection of the zarf on the ω, R plane for Blasius flow.

equation enables us to deduce a relation between α,β,ω,R which must be satisfied if ψ is nontrivial and which has proved useful as a correlator of transition.[1] In this connection ω is assumed to be real and in some sense prescribed; at each R, Eq. (2) determines a relation connecting α and β. Another relation is required[2-4] to complete the specification of α and β, namely that

$$\partial\alpha/\partial\beta=\sigma \tag{4}$$

is real.

We are then led to a convenient generalization of the classical concept of the neutral curve Γ_2 in two dimensions ($\beta=0$) to the zarf,[3] defined in the following way. For any real ω, Eqs. (2) and (4) define a neutral curve on which $\alpha_i=\sigma\beta_i$ and, in the cases tested[3,5,7,8] on this curve R takes a minimum value $R_z(\omega)$ at which $\alpha=\alpha_z$, $\beta=\beta_z$. The locus of α_z, β_z, and R_z as functions of ω defines the zarf. Equivalent definitions are: 1) the envelope of temporal neutral curves ($\omega_i=0$) obtained by varying α, R and holding α/β fixed; 2) the curve on which σ is real and α, β, ω are also real; and 3) the locus as a function of R of the extreme values of ω as α, β vary for fixed R, keeping α, β, and ω real.

The ideas embodied in Refs. 1-3 have since been utilized in Refs. 5-7 in connection with rotating disks (see also Ref. 8), yawed stagnation, and Blasius flows. With regard to the last two examples, it has been noticed[5,7] that when the base flow is of the form of Eq. (1) and is stable so that Γ_2 is closed as $R\to\infty$, the calculations of the zarf do not indicate that it closes in the (β,R) plane as $R\to\infty$. The purpose of this Note is to show that for such flows $|\beta|\to$ a finite limit on the lower branch of the zarf as $R\to\infty$.

We write

$$\alpha R=AR_2, \qquad \omega R=\omega_2 R_2, \qquad |\beta|=A \tag{5}$$

and let $R\to\infty$ holding A, R_2, ω_2 fixed. Then Eq. (2) reduces to

$$\psi''''-2A^2\psi''+A^4\psi=iR_2[(Au-\omega_2)(\psi''-A^2\psi)-Au''\psi] \tag{6}$$

i.e., the two-dimensional form. We now use the third equivalent definition of the zarf to obtain the values of A, R_2, ω_2. Then it is merely necessary to minimize ω_2R_2 on Γ_2. For example, in the case of Blasius flow and basing R on the displacement thickness, the minimum value of ω_2R_2 occurs at $R_2=703$ and $A=0.212$ and is 53.0. Hence on the lower branch of the zarf

$$R\alpha\to 149.1, \qquad |\beta|\to 0.212, \qquad R\omega\to 53.0 \tag{7}$$

This limit structure contrasts with that of Γ_2 in which

$$\alpha R^{1/4}\to 1.001(\lambda\delta_0)^{5/4}=0.497,$$
$$R^{1/2}\omega\to 2.229(\lambda\delta_0)^{3/2}=0.993 \tag{8}$$

where $\lambda=0.332$, and $\delta_0=1.721$.[9] It is interesting to note that the minimum value of ω_2R_2 occurs at the same solution set (0.0438, 0.1232, 703) of (ω, α, R) at which bifurcation of the zarf from Γ occurs.[5,7] The upper branch of the zarf coincides with that of Γ_2. In Fig. 1 we display the properties of the zarf for Blasius flow.

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